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Consistency simulation and optimization for HPIBM model in emergency decision making

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Abstract

Emergencies in ethnic regions in China happened increasingly, and emergency decision making become more complicated than that in other regions since it involves religions, cultures and national customs etc. The data consistency and incompleteness issues are two of the hot research topics in an emergency decision matrix (EDM). The Hadamard product induced bias matrix (HPIBM) model can be used to identify the most inconsistent entry in an EDM. In this paper, we provide experimental consistency simulations for HPIBM by randomly generating decision matrices with orders 3 to 7. In addition, we propose a novel HPIBM for estimating the missing judgments in an EDM by constructing optimization problem while improving the consistency ratio. A numerical example is used to illustrate the proposed method, and the result shows that the proposed method can find the optimal solutions for missing values and help emergency manager make fast response to the emergency incident even there are incomplete decision information.

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Keywords: Hadamard product induced bias matrix; simulation; emergency decision making; incomplete matrix; missing comparisons

1. Introduction

During the last decades, emergency happened increasingly around the world and emergency decision making has become one of the hot research topics in the study of decision making[1-4]. In recent years, emergencies in ethnic

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regions in China continue to occur, such as the "3.14 incident" in Tibet, "5.12" Wenchuan earthquake in year 2008, the Urumqi "7.5" riots in year 2009, Yushu earthquake in year 2010, and Ya'an earthquake in year 2013 etc. These incidents not only caused serious casualties and economic loss, but also affected the social stability and economic development in ethnic minority areas[5]. Different from the other regions in China, unconventional incidents in the ethnic regions have the following features such as concealment, complexity and situational characteristics, which make it more complicated and difficult to control[6-7]. For example, a regular emergency in ethnic area could easily evolve to an unconventional incident as long as it involves the national factors such as religion, culture and customs difference etc. Therefore, it is very important to study the emergency decision making in ethnic regions in China.

In the process of decision making for incident in the ethnic regions, two of the well-known multicriteria decision making methods, the Analytic Hierarchy Process (AHP)[8] and the Analytic Network Process (ANP)[9], can be used to identify the key scenario factors and make decisions[10-11]. However, as the three constraints in the emergency decision making, i.e. time constraint, limited information, and decision load constraint, decision makers may make inconsistent or incomplete judgments[12-14]. Therefore, the inconsistency and incompleteness issues of a pairwise comparison in emergency decision making become more serious than that in regular decision making by AHP and ANP[15-18]. To identify the most inconsistent entry in the original pairwise comparison matrix A, Kou et al.[19] proposed a Hadmard product induced bias matrix (in short HPIBM hereinafter) model. The mathematical theorem has been proved, and some numerical examples have been used to illustrate the proposed HPIBM model in Kou et al.[19].

In this paper, we aim to achieve two objectives by handling the experimental simulation problem for consistency and estimation problem for missing judgments. The first is to conduct experimental simulation on the HPIBM model for further validating the effectiveness of the HPIBM when it is used to improve the consistency. The simulations are conducted by generating randomly matrices with orders 3 to 7.

The second goal of this paper is to propose a novel HPIBM model for estimating the missing comparisons in an emergency decision matrix. Simply speaking, the missing comparisons in a matrix are first completed by unknown variables, then apply the HPIBM model to construct optimization problem to find the optimal solutions.

The rest of this paper is organized as follows. Next section first briefly describes the theorem of HPIBM, and then the experimental simulation and results are addressed. In Section 3, the HPIBM model is further extended to estimate the missing comparisons in an incomplete emergency decision matrix. A numerical example is used to illustrate the proposed method in this section. We conclude this paper in Section 4.

2. Experimental simulation

2.1. The theorem of Hadmard product induced bias matrix (HPIBM)

We first briefly present the theorem of HPIBM[19], and then conduct the consistency simulations by randomly generating pairwise comparison matrices. According to the theorem of HPIBM, the induced bias matrix (IBM) C in eq. (1) with the equality symbol "=" will hold if the pairwise comparison matrix A is perfectly consistent. If matrix A is just approximately consistent, then the IBM C in eq.(1) with symbol "$\approx$" will hold. Otherwise, there is at least one element in each row (column) of matrix C that deviates significantly from 1.

$$C = \frac{1}{n} AA \circ A^T = \left(c_{ij}\right) = \left(\frac{1}{n}\sum_{k=1}^{n} a_{ik}a_{kj}a_{ji}\right)\begin{cases} = U & if \quad a_{ik}a_{kj}a_{ji} = 1 \\ \approx U & if \quad a_{ik}a_{kj}a_{ji} \approx 1 \end{cases} \tag{1}$$

where $c_{ij} = \frac{1}{n}\sum_{k=1}^{n} a_{ik}a_{kj}a_{ji}$; $U = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$; n denotes the order of A; and A^T is the transpose of matrix A.

The symbol '$\circ$' denotes Hadamard product, *e.g.* $C = A \circ B$ indicating $c_{ij} = a_{ij}b_{ij}$ for all i and j.

The above theorem is proved mathematically and illustrated by some numerical examples in Kou et al.[19], interested readers are referred to Kou et al.[19] for details. In the following, we conduct the consistency simulations for random emergency decision matrices when the HPIBM is used to identify the key scenario factors of emergency incident in ethnic regions in China.

2.2. The experimental simulations and results

To validate the correctiveness of the HPIBM model for improving the consistency when applying it to emergency decision matrices, we conducted experimental simulations on inconsistency identification and adjustment by randomly generating 10^5 set of reciprocal pairwise comparison matrix (PCM) with orders 3 to 7. The random entries of the reciprocal PCM are subject to Saaty's 9-point scale and are randomly picked from the following numbers, 1/9, 1/8, /7, …, 1, 2, 3, …, 9. After generating the random PCM, the corresponding consistency ratio of each matrix is calculated, then apply the above HPIBM model to the matrices with CR>0.1 in order to identify and adjust the most inconsistent entry. The principle of identifying and adjusting the most inconsistent entry is to identify and adjust the largest value in the HPIBM C (say c^{max}_{ij}). If the consistency ratio of one random matrix cannot be reduced and lower than the threshold 0.1 by adjusting the corresponding entry a_{ij} in the random matrix A, then select the second, third largest entry in matrix C to improve its consistency ratio until it passes the consistency test.

The results of experimental simulations are obtained, as shown in Table 1. We can see from Table 1 that the numbers of random matrices for order 3 to order 7 with CR>0.1 are 79387, 96770, 99744, 100000 and 100000 respectively. Apply the HPIBM model to identify and adjust the most inconsistent entries in these matrices, we found that all the revised random matrices passed the consistency test and their consistency ratios have been improved and lower than 0.1. The corresponding simulation for matrices with different orders are plotted and shown in Figures 1-5.

Table 1. Simulation experiment for randomly generated matrices with different orders

Matrix Order	Number of simulation	Number of matrices with CR>0.1	Failed matrices	Passed Matrices
3	100000	79387	0	79387
4	100000	96770	0	96770
5	100000	99744	0	99744
6	100000	100000	0	100000
7	100000	100000	0	100000

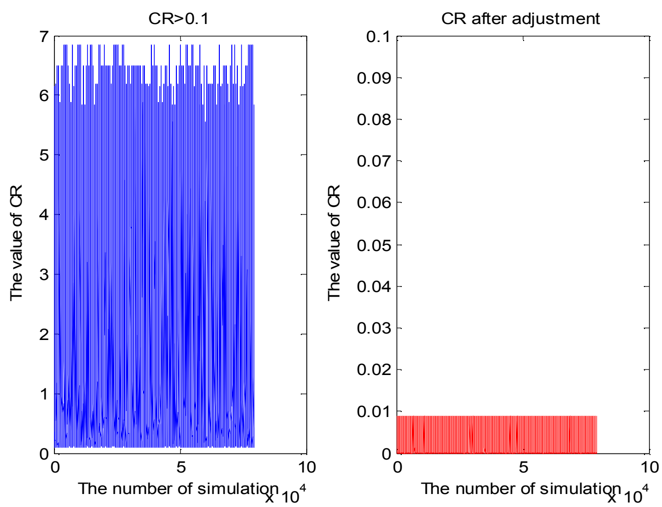

Fig.1 Experimental simulation experiment for 10^5 randomly generated matrices with order 3

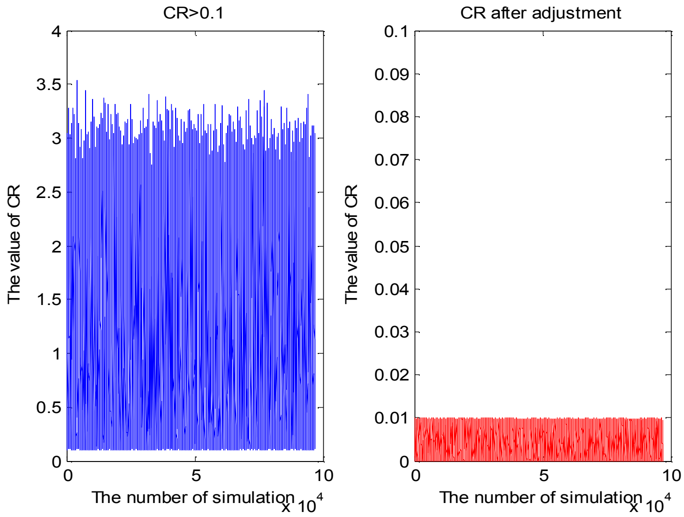

Fig.2 Experimental simulation experiment for 10^5 randomly generated matrices with order 4

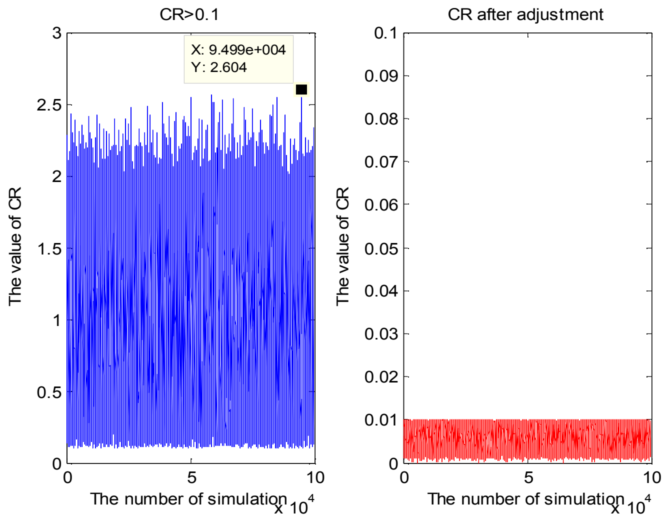

Fig.3 Experimental simulation experiment for 10^5 randomly generated matrices with order 5

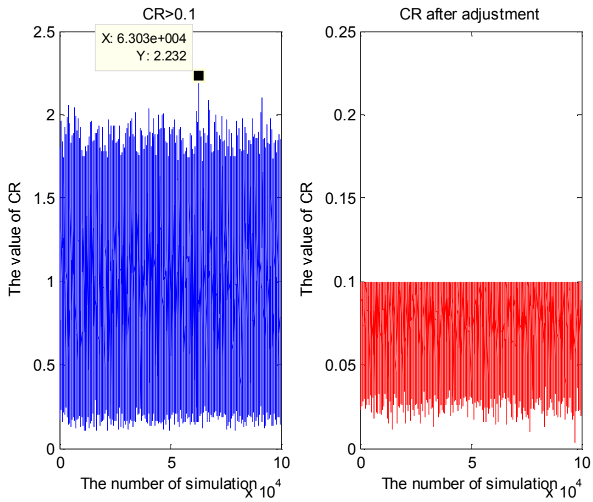

Fig.4 Experimental simulation experiment for 10^5 randomly generated matrices with order 6

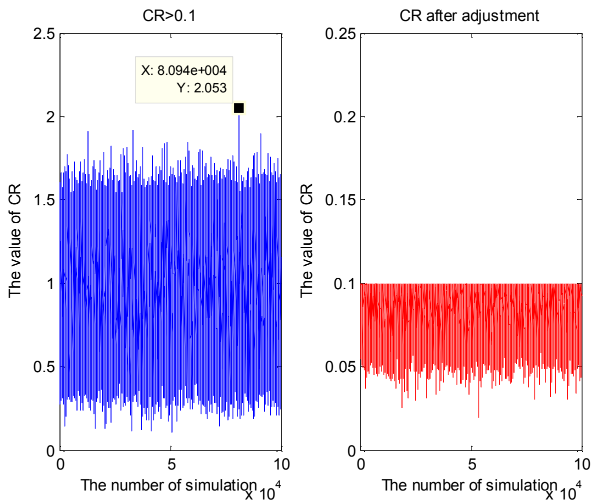

Fig.5 Experimental simulation experiment for 10^5 randomly generated matrices with order 7

3. HPIBM for estimating missing comparisons

3.1. The principle of HPIBM for missing comparisons

In the process of emergency decision making for the emergency incidents happened in the ethnic regions, decision makers may provide an incomplete decision making matrix because of the limited decision information, time limitation, religions and culture conflicts etc. Therefore, the missing comparisons in an emergency decision matrix (EDM) should be estimated before making a valid emergency decision. Based on the theorem of HPIBM model, we further extend it into the case of estimating the missing comparisons in an EDM.

Theorem 1: *The Hadarmad product induced bias error matrix (HPIBEM) ε should be equal (or close) to a zero matrix if judgment matrix A is perfectly (or approximately) consistent, that is,*

$$\varepsilon = \frac{1}{n} AA \circ A^T \text{ - } U = \left(c_{ij} - 1\right) = (\frac{1}{n}\sum_{k=1}^{n} a_{ik} a_{kj} a_{ji} - 1)\begin{cases} = 0 & if \quad a_{ik} a_{kj} a_{ji} = 1 \\ \approx 0 & if \quad a_{ik} a_{kj} a_{ji} \approx 1 \end{cases} \tag{2}$$

where A and A^T denote the revised 'complete' matrix A with unknown variables x_1 and $1/x_1$; x_2 and $1/x_2$; etc, and the corresponding transpose matrix respectively.

Proof: For the case that judgment matrix A is perfectly (or approximately) consistent, Kou et al.[19] have proved that $c_{ij} = \frac{1}{n}\sum_{k=1}^{n} a_{ik} a_{kj} a_{ji} = 1$. Obviously, we can obtain that,

$$\varepsilon_{ij} = \frac{1}{n}\sum_{k=1}^{n} a_{ik} a_{kj} a_{ji} - 1 = \begin{cases} = 0 & if \quad a_{ik} a_{kj} a_{ji} = 1 \\ \approx 0 & if \quad a_{ik} a_{kj} a_{ji} \approx 1 \end{cases}.$$

□

According to the above HPIBM for missing comparisons, we can estimate the missing values by constructing the optimization problem and minimizing the error matrix ε_{ij}, that is,

$$\begin{aligned} & Min \quad \varepsilon\left(a_{ij}, x\right) = \left(\left|\varepsilon_{ij}\left(x_1, x_2, \cdots x_p, a_{ij}\right)\right|\right) = \left|\frac{1}{n}\sum_{k=1}^{n} a_{ik} a_{kj} a_{ji} - 1\right| \\ & s.t. \quad 1/9 \le x \le 9 \end{aligned} \tag{3}$$

where a_{ik}, a_{kj}, a_{ij} may contain unknown variables x_1, x_2 etc. if the corresponding entry is missing.

In practice, the least square method (LSM) can be used to the above optimization problem, then we can construct the following one objective function to find the optimal solutions,

$$\begin{aligned} & Min \quad f\left(a_{ij}, x\right) = \sum_{j=1}^{n}\sum_{i=1}^{n}\left(\varepsilon_{ij}^2\left(a_{ij}, x\right)\right) = \sum_{j=1}^{n}\sum_{i=1}^{n}\left(\frac{1}{n}\sum_{k=1}^{n} a_{ik} a_{kj} a_{ji} - 1\right)^2 \\ & s.t. \quad 1/9 \le x \le 9 \end{aligned} \tag{4}$$

To summarize, the processes of estimating the missing comparison in an incomplete EDM by HPIBM model include:

Step 1: Construct the "Quasi-complete matrix" by pairs of unknown variables.
Step 2: Establish optimization problem by HPIBM model.
Step 3: Solve the optimal solution of the optimization problem.
Step 4: Test the consistency of the revised comparison matrix.

In the following section, a numerical example is used to illustrate the proposed method.

3.2. Numerical example

Assume there are five emergency responses alternatives, denoted as E1, E2, E3, E4 and E5, and a emergency expert provided the following emergency decision matrix A with maximum eigenvalue *5.0302* and *CR=0.0067*, which is already less than the threshold 0.1. To illustrate the processes of estimating the missing comparisons by HPIBM, suppose the following five comparisons are missing, i.e., a_{14}, a_{23}, a_{25}, a_{34}, a_{45}, and the incomplete EDM is written as $A(\times)$,

$$A = \begin{pmatrix} 1 & 6.2575 & 9 & 2.2817 & 2.4545 \\ 0.1598 & 1 & 1.7778 & 0.3333 & 0.3333 \\ 0.1111 & 0.5625 & 1 & 0.1429 & 0.1667 \\ 0.4383 & 3 & 7 & 1 & 1.0808 \\ 0.4074 & 3 & 6 & 0.9252 & 1 \end{pmatrix}, A(\times) = \begin{pmatrix} 1 & 6.2575 & 9 & \times & 2.4545 \\ 0.1598 & 1 & \times & 0.3333 & \times \\ 0.1111 & \times & 1 & \times & 0.1667 \\ \times & 3 & \times & 1 & \times \\ 0.4074 & \times & 6 & \times & 1 \end{pmatrix} \tag{5}$$

Apply the proposed HPIBM model to this matrix and follow the above four steps, we can find the optimal solutions. Details are shown below.

Step 1: Construct the "Quasi-complete matrix B" by filling in the missing comparisons with unknown variables $x_i (i=1,..5)$ and their reciprocals, we have ,

$$B = \begin{pmatrix} 1 & 6.2575 & 9 & x_1 & 2.4545 \\ 0.1598 & 1 & x_2 & 0.3333 & x_3 \\ 0.1111 & 1/x_2 & 1 & x_4 & 0.1667 \\ 1/x_1 & 3 & 1/x_4 & 1 & x_5 \\ 0.4074 & 1/x_3 & 6 & 1/x_5 & 1 \end{pmatrix} \tag{6}$$

Step 2: Establish optimization problem by HPIBM model.

$$Min \quad f(b_{ij},x) = \sum_{j=1}^{5}\sum_{i=1}^{5}\left(\varepsilon_{ij}^{2}(b_{ij},x)\right) = \sum_{j=1}^{5}\sum_{i=1}^{5}\left(\frac{1}{5}\sum_{k=1}^{5} b_{ik}b_{kj}b_{ji} - 1\right)^2 \tag{7}$$
$$s.t. \quad 1/9 \le x_i \le 9$$

Step 3: Find the optimal solutions of the above optimization problem.

For this optimization problem, we apply the *fmincon* (algorithm: 'medium-scale: SQP, Quasi-Newton, line-search') in Matlab software to estimate the optimal solutions. The results of optimization are shown in Table 2, and plotted in Figure 6. Figure 6(a) shows that the aggregation values of objective function *f(x)* reduced slightly after the 11th iteration (10th dot in the *x* axis) and stopped after 26 times of iterations. More precisely, Table 2 shows that aggregation values of objective function *f(x)* is less that 1 after the 11th iteration, and become almost stable after the 19th iterations. The estimated optimal solutions of five unknown variables, a_{14}, a_{23}, a_{25}, a_{34}, a_{45} are 2.0859, 1.6952, 0.3330, 0.1967, and 0.9986 respectively. These optimal solutions are close to the corresponding original values 2.2817, 1.7778, 0.3333, 0.1429 and 1.0808.

Step 4: Test the revised complete matrix by the optimal solutions. We can get that the $\lambda_{max} = 5.01621$, and the $CR=0.0036$, indicating the revised complete matrix passed the consistency test.

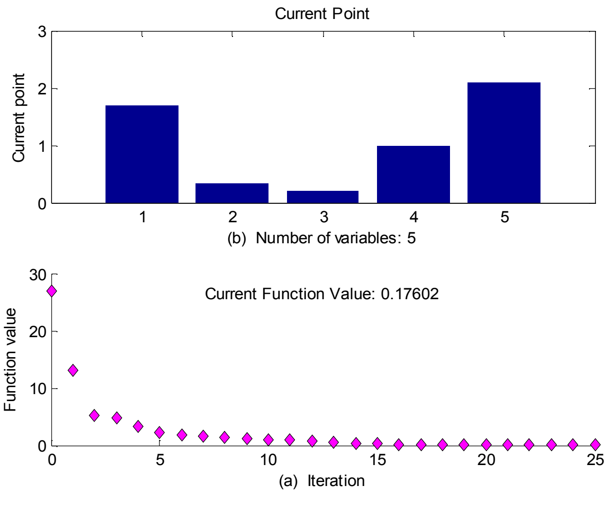

Fig. 6 Changes of objective functions and the estimated optimal solution

Table 2. The aggregation values of objective functions and the optimal solution

Iter	f(x)	Iter	f(x)	Iter	f(x)	Iter	f(x)	Iter	f(x)	variables x_i	Estimated values
0	26.8308	6	1.7052	12	0.72312	18	0.177406	24	0.176021	x_1	2.0859
1	13.0352	7	1.6415	13	0.63197	19	0.176564	25	0.176020	x_2	1.6952
2	5.2389	8	1.3479	14	0.41678	20	0.176159			x_3	0.3330
3	4.8343	9	1.1345	15	0.39218	21	0.176059			x_4	0.1967
4	3.3786	10	0.9925	16	0.19531	22	0.176036			x_5	0.9986
5	2.2399	11	0.8681	17	0.18149	23	0.176029				

It can be seen from the experimental results that the five assumed missing comparisons can effectively be estimated by the novel HPIBM model. In addition, the estimated values are very close to the original values in matrix A. When replacing the missing values by these estimated values, the revised matrix successfully passed the consistency test, therefore, the novel HPIBM for missing comparisons is effective and efficient. To make a comparison with the commonly used method in the Super Decision making software (in short SD), we calculated the priority vectors for the incomplete matrix by SD software, then applying the estimating formula $x_{ij}=\omega_i/\omega_j$ to estimate the missing comparisons. The comparison results are shown in Table 3. Obviously, although the estimated values by both methods are close to each other, except x_1, the values of x_2, x_3, x_4 and x_5 estimated by HPIBM method are closer to the original values than the values estimated by SD method. Therefore, the proposed method is accurate and efficient.

Table 3. The comparison results of the estimated missing comparisons by HPIBM method and SD method

Unknown variables	Priority vectors (ω_i)	SD method	HPIBM model	Original values
x_1	0.4562	2.1891	2.0859	2.2817
x_2	0.0706	1.6267	1.6952	1.7778
x_3	0.0434	0.3388	0.3330	0.3333
x_4	0.2084	0.2083	0.1967	0.1429
x_5	0.2214	0.9413	0.9986	1.0808

4. Conclusions

The inconsistency and incompleteness of an emergency decision matrix becomes more serious in ethnics regions because of the national factors such as religion conflicts, culture differences and national customs etc. In this paper, the consistency simulation is conducted to validate the capability of Hadamard product induced bias matrix (HPIBM) model for improving the consistency ratios. To simulate all possible cases of emergency decision matrices provided by emergency expert, 10^5 numbers of reciprocal pairwise comparison matrices with orders 3 to 7 are randomly generated, and random matrices with CR>0.1 are used to test the proposed HPIBM model. The experimental results show that all revised matrices with orders 3 to 7 passed the consistency test and their consistency ratios have been improved. In addition to experimental simulation, the HPIBM model is extended to estimate the missing comparisons in an incomplete emergency decision matrix. Briefly speaking, the missing comparisons are filled in by unknown variables, then apply the HPIBM model to the "quasi-complete" matrix, finally construct optimization problems by least square method to find the optimal solutions. The proposed HPIBM is an effective way to deal with the inconsistency and incompleteness issues in an emergency decision matrix.

Although the experimental simulation and numerical example for estimating missing comparisons validated that the HPIBM is an effective and accurate model for improving consistency ratio and estimating missing comparisons in an EDM, the simulation for random matrices with higher orders and the case study in real world emergency decision making are our next research directions.

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